II. *De Inventione Centri Oscillationis. Per* Brook Taylor *Armig. Regal. Societat. Sodal.*

Definitio.

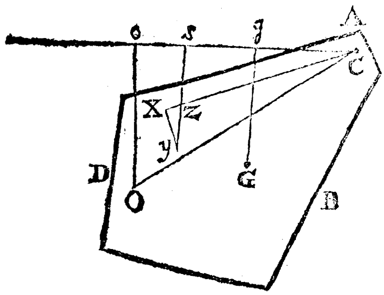
Est Centrum Oscillationis punctum quoddam in corpore pendulo, cujus vibrationes singulæ eodem modo atq; eodem tempore peraguntur, ac si illud solum ad eandem distantiam a puncto suspensionis filo suspenderetur.

PER se vix satis manifestum est in corpore aliquo dari hujusmodi punctum: utpote cujus acceleratio debeat, (*per hanc def.*) in omnibus inclinationibus corporis penduli ad Horizontem, perinde esse, ac si a propriâ tantùm gravitate urgeatur; reliquis particulis totius corporis ejus motum proprium haud perturbantibus. Itaq; in ordine ad inventionem hujus Centri, præmittenda est una atq; altera propositio, unde constet tale punctum dari.

Prop. 1. Prob. 1

In corporis Oscillantis datâ quâvis inclinatione ad Horizontem, invenire punctum cujus acceleratio perinde sit, ac si ab ipsius propriâ tantum gravitate urgeatur.

Sit A B D corporis propositi sectio in plano ad Horizontem perpendiculari, in quo movetur centrum gravitatis G, centro suspensionis existente C. Distinguatur corpus in elementa prismatica plano A B D perpendicularia,



cularia, adeoque Horizonti semper parallela; ut facile patebit ex motu centri gravitatis G in plano illo A B D. Atq; ob hujusmodi situm, tale elementum quodvis spectari potest tanquam punctum Physicum p in plano eodem A B D ad punctum z locatum. Reducatur itaq; corpus propositum in planum Physicum A B D constans ex hujusmodi particulis p.

In hoc plano ut inveniatur punctum O, cujus acceleratio propria non mutatur ab actionibus particularum reliquarum, attendendum est ad vires particulæ cujusvis singularis p in puncto z sitæ. Nam ex hisce viribus conjunctis oritur plani totius motus absolutus; cujus ope datur motus puncti cujusvis propositi; unde vicissim invenitur punctum cujus motus est datus.

At urgetur particula p a vi propriæ gravitatis; quæ si partium cohæsio dissolveretur, in dato tempore minimo, datam produceret accelerationem motûs in perpendiculari ad Horizontem zy. Ad Cz duc normalem yx, & resolvetur acceleratio zy in partes zx & xy. Ob corporis rigiditatem, tollitur vis zx per resistentiam puncti C. At vi reliquâ xy trahitur spatium ABD in gyrum circa punctum C; & ductâ horizontali Co & perpendiculari zs, erit ea ut $\frac{Cs}{Cz}$: Nempe ob gravitatis vim datam, & similia triangula xyz & sCz. Ergo vis particulæ p ad movendum spatium ABD est ut $\frac{Cs}{Cz} \times p$.

Ad has vires in unum colligendas, sit O punctum invariabile, in lineâ ad libitum ducta & ad distantiam adhuc incognitam CO. Tum erit vis particulæ p ad movendum

vendum punctum O, ut $\frac{Cz}{CO} \times \frac{Cs}{Cz} \times p$, hoc eſt ut $\frac{Cs}{CO} \times p$. Acceleratio autem, quam tribuit p eidem puncto O, erit ut $\frac{CO}{Cz} \times \frac{Cs}{Cz}$. Itaq; applicatâ vi illâ $\frac{Cs}{CO} \times p$ ad hanc accelerationem $\frac{CO \times Cs}{Czq:}$, erit quotiens $\frac{Czq:}{COq:} \times p$ particula, quæ, ſi in ipſo puncto O fingatur moveri cum eâdem acceleratione $\frac{CO \times Cs}{Czq:}$, eundem omnino produceret motum, quem in eodem puncto O producit particula p. Hinc demum reducitur Problema ad motuum Theorema notiſſimum: Applicatâ enim ſummâ virium $\frac{Cs}{CO} \times p$ ad ſummam particularum $\frac{Czq:}{COq:} \times p$, erit quotiens acceleratio abſoluta puncti O. Dein ductâ perpendiculari O o, & poſitâ hac acceleratione æquali datæ accelerationi $\frac{Co}{CO}$ ipſius puncti O, dabitur diſtantia CO. Sit enim $\frac{Co}{CO} = d$, & (juxta methodum *Fluxionum*) $Cs \times p = \dot{M}$, & $Czq: \times p = \dot{C}$. Tum ob CO invariabilem erit ſumma omnium virium $\frac{Cs}{CO} \times p = \frac{M}{CO}$, & ſumma omnium particularum $\frac{Czq:}{COq:} \times p = \frac{C}{COq:}$. Unde, applicatâ ſummâ momentorum ad ſummam corporum, erit $\frac{M}{C} \times CO = d$ adeoq; $CO = \frac{dC}{M}$. Inventis igitur C & M, per *Fluxionum* methodum inverſam, dabitur C O. Q. E. I.

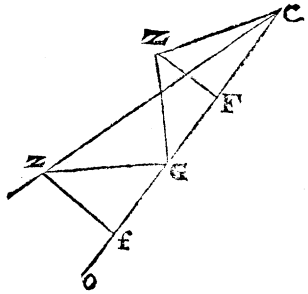
Cor. A centro gravitatis G ad horizontalem C o duc perpendicularem G g, & ſit corpus ipſum A B C = A. Tum

Tum ex notissimâ indole centri gravitatis erit $M = Cg \times A$.
Unde est $CO = \frac{dC}{Cg \times A}$.

Prop. 2. Theor. 1.

Iisdem positis, quæratur punctum O in rectâ C G transe-

$= CGq: + Gzq: + 2CG \times Gf$: Est ergo $C =$ (aggregato omnium $Czq: \times p =$) aggregato omnium $CGq: \times p + Gzq: \times p - 2CG \times GF \times p + 2CG \times Gf \times p$. At ob centrum gravitatis G, est aggregatum omnium $2CG \times GF \times p =$ aggregato omnium $2CG \times Gf \times p$. Quare est $C =$ aggregato omnium $CGq: \times p + Gzq: \times p = CGq: \times A + D$. At enim *per Theor.* 1. est $CO = \frac{C}{CG \times A}$. Ergo $CO = CG + \frac{D}{CG \times A}$. Q. E. D.



Cor. Hinc datur parallelogrammum $CG \times GO$. Est enim $GO = \frac{D}{CG \times A}$. At dantur A & D. Quare datur $CG \times GO = \frac{D}{A}$.

Prop. 4. Theor. 3.

Iisdem positis, si in puncto O constituatur particula physica $\frac{CG \times A}{CO}$, *quæ propria gravitate agitata Oscillet circa punctum C; spatij A B C motus perinde omnino erit, ac si agitaretur ab Oscillatione ipsius corporis A.*

Constat tam ex Natura centri gravitatis, quam *per Prob* 1. Est enim $\frac{CG \times A}{CO}$ aggregatum omnium $\frac{Czq: \times p}{COq:} = \frac{C}{COq:}$.

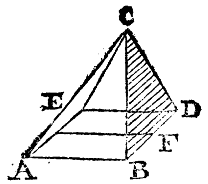
Prop. 5. Prob. 2.

Datis corporis cujusvis magnitudine A, centro gravitatis G, & puncto suspensionis C. Invenire ejusdem centrum Oscillationis O.

Fit per Theor 1. inveniendo quantitatem C; vel per Theor. 2. quærendo quantitatem D.

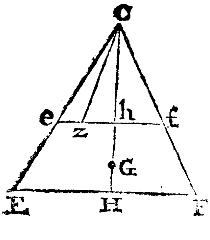
Scholium.

Ad instituendum calculum in casu particulari, eligenda est quantitas C vel D, prout suggerit natura figuræ propositæ. Dein datâ earum alterutrâ, altera item dabitur per æquationem (*Prop.* 3.) $C = CGq : \times A + D$. Unde etiam dabitur pgr. $CG \times GO = \frac{D}{A}$ (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq$:. Cujus ope, ex datis centro gravitatis & puncto suspensionis, datur centrum Oscillationis per solam divisionem. Quare in quolibet exemplo semper commodissimum erit hoc parellelogrammum primùm eruere, vel per computum ipsius D, vel per quantitatem C, ex idoneâ assumptione centri suspensionis.



Superest, ut hæc exemplis aliquot illustremus.

Ex. 1. Sit figura proposita Pyramis A D C, cujus basis est pgr. A D, sitque motus centri gravitatis in plano transeunte per verticem C & diametrum basis E F lateri A B parallelam.



Ad calculum commodissime instituendum, sit ipse vertex C centrum suspensionis. Tum ad modum *Prob.* 1. reducatur figura ad planum physicum trianguli Isoscelis C E F, in quo e f parallela ipsi EF repræsentat lineam physicam ex particulis p compositam. Sit C H = a.

H F

$HF = b$, & $Ch = x$. Tum ex naturâ figuræ erit $eh = \frac{bx}{a}$, & particula p sita ad punctum z erit ut x; vel potius, facto $hz = v$, erit $\dot{v}\dot{x}$ elementi prismatici basis, & p erit ut $\dot{v}\dot{x}x$. Unde erit $\dot{C} = Czq: \times \dot{v}\dot{x}x = \dot{v}xx^3 + \dot{x}\dot{v}v^2x$. Ideoq; summa omnium $Czq: \times p$ in lineâ hz erit $v\dot{x}x^3 + \frac{\dot{x}xv^3}{3}$; & in lineâ ef (pro v ponendo $\frac{bx}{a}$) erit summa illa $\frac{6ba^2 + 2b^3}{3a^3} \times \dot{x}x^4$. Unde iterum capiendo fluentem, & pro x scribendo a, erit $C = \frac{6ba^2 + 2b^3}{15} \times a^2$. Est autem pyramis ipsa $A = \frac{2baa}{3}$, & distantia centri gravitatis G a vertice C est $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq: = \frac{D}{A} = CG \times GO = \frac{3a^2 + 16b^2}{80}$.

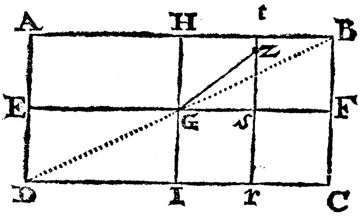
Ex. 2. Sit figura proposita Conus rectus descriptus rotatione trianguli isoscelis ECF circa perpendiculum CH.

Hic iterum sumpto vertice C pro centro suspensionis, & factis $CH = a$, $HE = b$, $Ch = x$, $hz = v$, ut supra; erit $p = 2\dot{x}\dot{v} \times \sqrt{\frac{bb}{aa}xx - vv}$; unde $\dot{C} = 2\dot{v}\dot{x} \times \overline{xx + vv} \times \sqrt{\frac{bb}{aa}xx - vv}$. Sit B segmentum circuli diametro ef descripti, quod adjacet Abscissæ $hz = v$,

& Ordinatæ $\sqrt{\frac{bb}{aa}xx - vv}$; tum erit ſumma omnium $Czq : \times p$ in rectâ $hz = 2\dot{x} \times \frac{4a^2 + b^2}{4a^2} x^2 B - \frac{1}{3}\dot{x} v \times \overline{\frac{b^2}{a^2}x^2 - v^2}|^{\frac{3}{2}}$. Et quando $v = e b$, erit hæc ſumma $2\dot{x} \times \frac{4a^2 + b^2}{4a^2} x^2 B$; cujus duplum $\frac{4a^2 + b^2}{a^2} \dot{x} x^2 B$ eſt pars ipſius C in rectâ e f. Eſt autem area B ut x^2; ſit ergo $B = c x^2$; atq; pars illa ipſius C erit $\frac{4a^2 + b^2}{a^2} \times c \dot{x} x^4$. Unde capiendo fluentem erit $C = \frac{4a^2 + b^2}{5} \times c a^3$. Eſt autem conus ipſe $A = \frac{4}{3} c a^3$, & $CG = \frac{3}{4} a$. Unde $\frac{C}{A} - CGq : = \frac{D}{A} = \frac{3a^2 + 12b^2}{80}$.

Atq; ad hunc modum procedit calculus in aliis figuris, ubi rationes C h ad h e, & h z ad p ſunt magis compoſitæ.

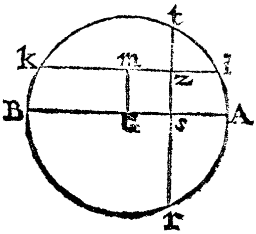
Ex. 3. Ut pateat ratio calculi quantitatis D, ſit figura propoſita parallelepipedon, cujus facies Horizonti perpendicularis, & parallela plano motûs centri gravitatis eſt A B D. Duc diametros E F & H I, & ſit altitudo elementorum p, :: & ſit t r parallela H I; & $GF = a$, $GH = b$, $Gs = x$, & $sz = v$. Tum erit $\dot{D} = \dot{v}\dot{x}xx + \dot{x}\dot{v}vv$. Unde ipſius D pars in rectâ t r erit $2b\dot{x}x^2 + \frac{2}{3}b^3\dot{x}$: atq; iterum ſumendo fluentis duplum, erit



D

$D = \frac{4 b a^3 + 4 b^3 a}{3}$. Atqui est $A = 4 a b$; unde est $\frac{D}{A} = \frac{aa + bb}{3} = \frac{1}{12}$ D B quad.

Ex. 4. Sit ultimum exemplum in Sphærâ, cujus circulus maximus B t r, diameter A B, & centrum G. Tum ductis lineis ut in Schemate satis patent, erit $\dot{D}$ = G s q: × p + Gm q: × p. At summa omnium G s q: × p in recta t r est G s q: ductum in aream circuli diametro t r descripti. Item summa omnium G M q: × p in rectâ k i est G m q: × aream circuli diametro k i descripti. Unde statim constat esse D = quater fluenti ipsius G s q: in aream circuli cujus diameter est t r. Sit ergo c area circuli cujus radij quadratum est 1, & sit G A = a, & G s = x. Tum erit $\dot{D} = 4 \dot{x} x x \times c a a - c x x = 4 c a^2 \dot{x} x^2 - 4 c \dot{x} x^4$. Unde sumendo fluentem, & faciendo x = a, erit $D = \frac{8}{15} c a^5$. Est autem $A = \frac{4}{3} c a^3$.



Unde $\frac{D}{A} = \frac{2}{5} a a$.

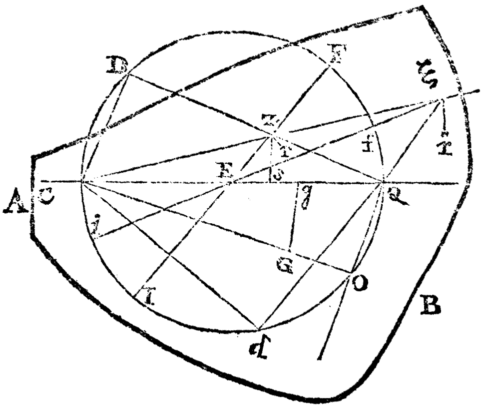
Ob affinitatem solutionis libet his subjungere Problema de inventione Centri Percussionis.

Prop. 6. Prob. 3.

Corporis cujusvis circa datum punctum rotati, invenire Centrum Percussionis; punctum scilicet tale, ut Corpus in illud impingens, & eâdem operâ solutum a puncto suspensionis, neque huc neque illuc inclinet.

Primùm constat hoc punctum quæri debere in plano motûs centri gravitatis. Si enim corpus resolvatur in e-

 lementa



lementa prismatica plano isti normalia, ferentur ea motu sibi parallelo; unde momenta ex utrâq; parte istius plani erunt æqualia; adeoq; per resistentiam factam in hoc plano, corporis punctum nullum de eo pelletur. Sit ergo planum illud A B, ad quod reducetur corpus per contractionem elementorum prismaticorum in particulas p ad puncta z sitas, ut in *Prob.* 1. In hoc plano sit C centrum rotationis; aut saltem ejus projectio facta per lineam perpendicularem in hoc planum demissam; & sit Q punctum quæsitum. Per C duc ad libitum C ξ, in quâ sume puncta duo z & ξ, ita ut ductis z Q & ξ Q, sit angulus C z Q obtusus, & angulus C ξ Q acutus: atque in punctis z & ξ sint particulæ p & π. Tum ad C ξ ductis normalibus z r & ξ *r*, quæ sint ad invicem ut C z ad C ξ, ijs repræsentabuntur velocitates absolutæ particularum p & π. At harum velocitatum partes quæ sunt in directionibus z Q & ξ Q, tolluntur per resistentiam puncti Q. Ad Q z & Q ξ duc normales C D & C d, & ob angulos æquales z C D = r z Q, & ξ C d = *r* ξ Q, velocitatum partes reliquæ, in directionibus ipsis Q z & Q ξ perpendicularibus, erunt ut z D & ξ d. Unde habitâ ratione distantiarum Q z & Q ξ, erunt vires particularum p & π ad movendum spatium A B in partes contrarias, ut D z × z Q × p, & d ξ × ξ Q × p. At per conditiones Problematis debent summæ hujusmodi contrariarum virium esse inter se æquales.

Ob angulos ad D & d rectos, sunt puncta D & d ad circumferentiam circuli diametro C Q descripti. Sit istius circuli centrum E. Tum ductis E z & E ξ circulo occurrentibus in F & I, f & i, erit $Dz \times zQ = Fz \times zI = EFq: - Ezq: = EQq: - Ezq:$, & $d\xi \times \xi Q = E\xi q: - EQq:$ Quare erit summa omnium $EQq: \times p - Ezq: \times p =$ summæ omnium $E\xi q: \times \pi - EQq: \times \pi$; & terminis transpositis, summa omnium $EQq: \times \overline{p + \pi}:$ = summæ omnium $Ezq: \times p + E\xi q: \times \pi$, hoc est, si p ponatur tam pro particulâ p intra circulum, quam pro particula π extra circulum, erit summa omnium $EQq: \times p =$ summæ omnium $Ezq: \times p$. Ad C Q duc normalem z s. Tum erit $Ezq: = Czq: + ECq: - QC \times Cs$. Quo valore ipsius $Ezq:$ ei substituto, & æquatione debité tractatâ, tandem invenies summam omnium $CQ \times Cs \times p =$ summæ omnium $Czq: \times p$. Unde est $CQ = \frac{\text{summæ omnium } Czq: \times p}{\text{summ: omnium } Cs \times p}$. At enim est summa omnium $Czq: \times p$ ipsa quantitas C in calculo centri Oscillationis: & si centrum gravitatis sit G, & ad C Q ducatur normalis G g, & corpus ipsum dicatur A, erit summa omnium $Cs \times p = Cg \times A$. Unde est $CQ = \frac{C}{Cg \times A}$. Sit centrum Oscillationis O; tum per *Theor.* 1. erit $CO = \frac{C}{CG \times A}$. Unde est $Cg : CG :: CO : CQ$. Quare per O ducta ad C O perpendicularis transibit per punctum Q. Q. E. I.

III. *Epistola*